Pythagorean Theorem

Background information

Pythagorean Theorem states that in any right-angled triangle, the area of the square whose side is the hypotenuse (the side opposite to the right side) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at the 90 degree angle).

Let $a$ and $b$ be the legs of a right-angled triangle. Let $c$ be the hypotenuse. Then the Pythagorean equation is $a^2 + b^2 = c^2$

Activity

Origami proof of the Pythagorean Theorem - by Vi Hart
http://www.youtube.com/watch?v=z6lL83wl31E

We do not need numbers or fancy equations to prove the Pythagorean Theorem, all we need is a piece of paper. Instead of looking at diagrams, we will be folding our piece of paper.

Directions

- **Step 1:** Acquire a paper square.
- **Step 1:** Fold it in half three times.
- **Step 2:** Fold parallel to the edges anywhere you choose and extend the crease.
- **Step 3:** Fold back four right triangles around the square and admire the area hypotenuse squared that is left over.
- **Step 4:** unfold and rip along a short side to fold back another four right triangles and admire the area one leg squared plus the other leg squared that is left.

Extra question

Think of a way you could convince yourself that no matter what the triangles on the outside look like, this will always be a square. Also, these edges look like they line up together. Do they always do that? Is it exact?

Full transcript of the video

- **Step 1:** Fold your square in half one way, then the other way, then across the diagonal. No need to make these creases sharp, we're just taking advantage of the symmetries of the square for the next step. But, be precise.
- **Step 2:** Make a crease along this triangle, parallel to the side of the triangle that has the edges of the paper. You can make it anywhere you want. This is where you are choosing how long and pointy, or short and fat, your right triangle is going to be, because this is a general proof. Now when you unwrap it, you will have a square centered in your square. Extend those creases and make them sharp, and now we've got four lines all the same distance from the edges, which will allow us to make a bunch of right triangles that are all exactly the same.
- **Step 3:** Fold from this point to this one. Basically taking a diagonal of this rectangle. Now we've got our first right triangle. Which has the same shape and area as this one. Let's call the sides: "A little leg", "a big leg", and "hypothenus". Rotate ninety degrees, and fold back another triangle, which of course is just like the first. Repeat on the following two sides. The original paper minus those four triangles, gives us a lovely square. How much paper is this? Well, the length of a side is the hypotenuse of one of these triangles. So the area is the hypotenuse squared.
Step 4: Unfold, and this time let's choose a different four triangles to fold back. Rip along one little leg, and fold back these two triangles. Then you can fold back another two over here. The area of the unfolded paper, minus four triangles, must be the same, no matter which four triangles you remove. So let's see what we've got. We can divide this into two squares, This one has sides the length of the little leg of the triangle. And this one has sides as long as the big leg. So the area of both together, is little leg squared, plus big leg squared. Which has to be equal to this area, which is hypotenuse squared. If you called the sides of your triangle something more abstract, like: a, b, and c, you'd of course have a squared plus b squared equals c squared

Selected Curriculum Links

Grade 8
http://www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf
- “Geometry and Spatial Sense: sorting quadrilaterals by geometric properties involving diagonals; constructing circles; investigating relationships among similar shapes; determining and applying angle relationships for parallel and intersecting lines; relating the numbers of faces, edges, and vertices of a polyhedron; determining and applying the Pythagorean relationship geometrically; plotting the image of a point on the coordinate plane after applying a transformation”.
- “Determine the Pythagorean relationship, through investigation using a variety of tools (e.g., dynamic geometry software; paper and scissors; geoboard) and strategies; solve problems involving right triangles geometrically, using the Pythagorean relationship”
- “Pythagorean relationship. The relationship that, for a right triangle, the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on the other two sides”

Grade 9 and Grade 10
- Section on “Solving Problems Involving Perimeter, Area, Surface Area, and Volume”
- Section on “Solving Problems Involving Perimeter, Area, and Volume”
- Section on “Solving Problems Involving the Trigonometry of Acute Triangles”
- Section on “Solving Problems Involving the Trigonometry of Right Triangles”

Grade 11 and Grade 12
- “Prove simple trigonometric identities, using the Pythagorean identity”
- “Construct accurate right angles in practical contexts (e.g., by using the 3-4-5 triplet to construct a region with right-angled corners on a floor), and explain connections to the Pythagorean theorem”

Resources

- More proofs of the Pythagorean Theorem
  - http://www.mathsisfun.com/pythagoras.html
  - https://www.khanacademy.org/math/geometry/right_triangles_topic/pyth_theor/v/the-pythagorean-theorem
- Proofs of the Pythagorean Theorem with interactive applets
  - http://www.ies.co.jp/math/java/geo/pythagoras.html